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A SIMPLE DIVERSIFIED PORTFOLIO STRATEGY

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We present a simple portfolio construction approach which is a blend of market weights and equal stock and sector weights. Our approach results in a highly diversified portfolio both on a stock level and on a sector level and generates higher portfolio returns at slightly lower risk than a market weighted index. We demonstrate that the higher returns of our diversified portfolio originate both from mitigating the link with market weights and from its higher return benefit due to diversification which we are able to capture because we rebalance our portfolio on a regular basis. Our diversified portfolio is highly implementable and has very high investment capacity.

1 Background

Over the last few decades market weighted (MW) indexing has become by far the predominant investment approach for capturing the broad equity risk premium. Although it offers low costs and turnover, high scalability and has historically enjoyed considerable theoretical backing, in recent years critics have pointed out significant drawbacks with MW portfolios.[1]

In this paper we *focus primarily on the issue of diversification* and argue that MW portfolios tend to be insufficiently diversified both on a stock level and on a sector level. As we will demonstrate, a lack of diversification can lead to lower portfolio returns. While diversification is most commonly analyzed in the context of risk reduction, an additional benefit of diversification is the return premium it generates for a portfolio that is rebalanced regularly.

To illustrate the lack of diversification Exhibit 1 shows aggregate stock and sector weights for an MW portfolio that consists of the 500 largest stocks in the US (a close approximation to the widely used S&P 500 index).

The graphs in Exhibit 1 show that market weights are very concentrated in mega-cap stocks. The top-10 stocks in terms of market weights (i.e. just 2% of the number of companies) have an average aggregate market weight of almost 20% over

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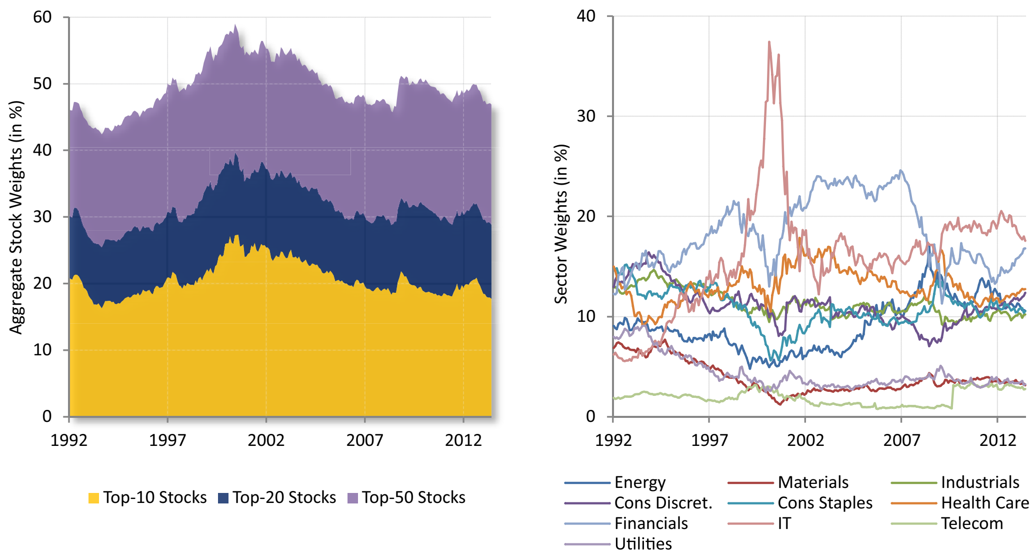
Exhibit 1 Top-500 market weighted US stocks—aggregate stock weights and GICS sector weights.



Source: Global Systematic Investors, based on Factset/Standard and Poors data.

our sample period from 1992 to 2013. Moreover, company membership in the top-10 list tends to change substantially over time which casts doubt on whether these companies should get such a high portfolio weight.[2]

Similarly, Global Industry Classification Standard (GICS) sector market weights are at times highly skewed towards certain individual sectors such as Information Technology during the technology boom at the end of the 1990s. At the time more than one third of the entire US market capitalization was concentrated in this single sector. Hence MW portfolios tend to be insufficiently diversified both on a stock level and on a sector level and allocate more to higher priced stocks and sectors. Moreover, levels of stock and sector concentration in MW portfolios can vary significantly through time.

Previous work has shown that there is a return premium that results from diversification as first documented by Booth and Fama (1992) on asset classes. Fernholz *et al.* (1998) illustrate this effect on a stock level and Leclerc *et al.* (2013) show it on a sector level. This return premium can be captured if a portfolio is rebalanced regularly in order to remain diversified (see, e.g., Willenbrock, 2011; Qian, 2012). Following these authors, we refer to this return premium as the *diversification return* and analyze it in more detail later on.

We show that the relative performance of a diversified portfolio compared to a passive MW portfolio depends on changes in market concentration and the portfolio's diversification return. If the market becomes more concentrated, this diminishes or may even overwhelm the portfolio's diversification return and the portfolio's relative return suffers. However, since the market cannot concentrate forever and increasing market concentration is unlikely to dominate the diversification return for long, the diversified strategy outperforms in the long run.

Moreover, as we will show, the fact that the market does not concentrate forever implies a negative relationship between market weights and subsequent stock returns in the long run. Strategies that mitigate the link between portfolio weights and market weights diminish this negative relationship and therefore enhance portfolio returns. This is true even for strategies that appear to contain no information such as monkeys picking stocks (see Arnott *et al.*, 2013; Clare *et al.*, 2013).

The simplest portfolio which is both well-diversified and mitigates the link between portfolio weights and market weights is an equal-weighted (EW) portfolio. Among others, Bouchey *et al.* (2012) and Plyakha *et al.* (2012) examine the appeal of equal-weighted portfolios. However, equal stock weights may hamper sector diversification (e.g. Velvadapu, 2011) and vice versa. Therefore, achieving both equal stock weights and equal sector weights would be desirable in principle but infeasible. Instead, however, we can shrink towards equal stock and sector weights as described in this study.

One issue with EW portfolios is that they exhibit extreme overweights in tiny stocks or sectors relative to MW portfolios. This is problematic because it can severely restrict the capacity of a strategy if small cap stocks are included in the investment universe. It may also result in prohibitively high trading costs as extreme positions in illiquid names need to be established. Therefore, rather than holding an EW portfolio on a stock or sector level, we blend equal stock and sector weights with market weights. This blended approach allows us to capture the best of both worlds, namely most of the benefits of EW portfolios outlined above and similar capacity and scalability as MW portfolios as well as similarly low turnover.

We demonstrate that a portfolio constructed in this manner exhibits a strong and consistent return premium which is due to its diversification return. The diversification return is not captured by the Fama–French–Carhart four-factor model (Fama and French, 1993; Carhart, 1997) and almost certainly underlies some of the return premium that alternative indexing strategies deliver.

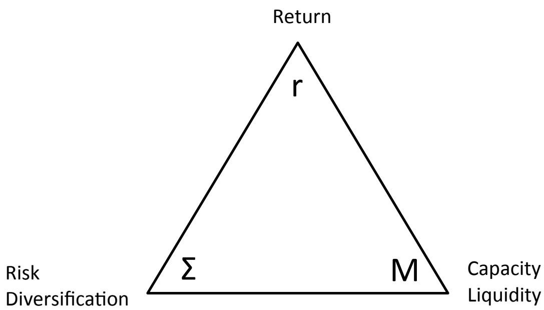
We make the following contributions to the existing literature: (1) We combine stock and sector-level diversification by shrinking from market weights towards equal stock and sector weights simultaneously; (2) We decompose equal-weighted relative portfolio returns into a change in market concentration term and a diversification return term; and (3) We demonstrate that there is a straightforward portfolio construction methodology that allows us to exploit these findings in a scalable, low turnover and therefore easily implementable portfolio.

Our simple diversification strategy is a superior way of capturing the equity risk premium compared to MW portfolios. It therefore establishes a tougher benchmark for active fund managers than MW indices as it results in well-diversified, high-capacity and low turnover portfolios that can be delivered very cheaply.

2 Portfolio construction framework

To construct a portfolio, we focus on investors' three key objectives namely: (1) high returns; (2) low risk or high diversification; and (3) high capacity and low trading costs. We illustrate these three dimensions in the triangle below. A complete and effective investment strategy needs to incorporate all three of these objectives simultaneously.

Portfolios or investment strategies positioned on the vertices of the triangle below address only one objective and ignore the two others. For example, if we ignore expected returns and risk and our only concern is low trading costs and a highly scalable portfolio, we would invest in a portfolio that is maximized for scalability and low turnover such as a market weighted index (shown as **M**). If we only care about higher returns but ignore trading costs and risk, we weight stocks by their expected returns only (shown as **r**). Finally, if we only care about risk and ignore return and trading costs, we should invest in a portfolio that purely minimizes risk (shown as **Σ**) which corresponds to the minimum variance portfolio.



The triangle above is a graphical illustration of an investor's utility function of the form[3]

$$U(w) = w'r - \frac{1}{2}\lambda w'\Sigma w - \frac{1}{2}\gamma w' M^{-1} w$$

where w is a vector of portfolio weights in each stock, Σ is a covariance matrix and M is a square matrix with market caps along its main diagonal and zeros elsewhere. As outlined in the Appendix in more detail, the above utility function has three terms: (1) an expected return term ($w'r$), (2) a risk term ($1/2\lambda w'\Sigma$w) and (3) a trading cost and capacity term ($1/2\gamma w' M^{-1} w$). The parameters λ and γ depend on the investor's risk and trading cost aversion.

The above utility function is maximized when the following holds

$$w = (\lambda\Sigma + (1-\lambda)M^{-1})^{-1} r \tag{1}$$

where, without loss of generality, we have set $\lambda + \gamma = 1$ and $0 \le \lambda \le 1$. We can derive the MW portfolio and all the standard alternative indexing methodologies from this formula.[4,5]

If an investor only cares about trading costs and scalability and ignores expected stock returns (in this case λ is set to zero and we can replace r with ι, a vector of ones) then Equation (1) becomes $M\iota$ which is simply the MW portfolio.

If an investor does not care about trading costs (in this case λ is set to 1) then Equation (1) becomes $\lambda\Sigma^{-1} r$ which is the solution to a standard mean–variance optimization that only takes expected return and risk into account and ignores trading costs. These portfolios would lie on the top left-hand side of the triangle. By replacing r with ι (a vector of ones) then Equation (1) becomes $\lambda\Sigma^{-1}\iota$ which is the solution to an unconstrained minimum variance optimization which lies at the bottom left vertex of the triangle above. Other risk-based variants can easily be generated by modifying the specification of Σ. For example, EW strategies are generated using $\lambda\Sigma^{-1}\iota$ by setting Σ to a simple $N \times N$ matrix (if N stocks are in our universe) with N along its main diagonal and zeros elsewhere.

If the investor does not care about risk but only about achieving higher returns and scalability ($\lambda = 0$), then Equation (1) becomes Mr which corresponds to a fundamentally weighted portfolio such as Fundamental Indexing™. This relationship has been outlined by Asness (2006).[6]

In this study we focus on the benefits of (1) reducing the negative correlation between portfolio weights and subsequent stock returns which can be observed for MW portfolios, and (2) improved diversification compared to an MW portfolio. We are therefore agnostic about expected asset returns (r).[7] Instead we construct a portfolio with similar characteristics as an MW index but which mitigates its drawbacks. We can achieve this by replacing r with ι (a vector of ones) in Equation (1) which results in

$$w = (\lambda\Sigma + (1-\lambda)M^{-1})^{-1}\iota$$

The set of portfolios defined by this equation are located on the lower side of the triangle as return views are not incorporated.

This alternative portfolio is a blend of market weights and a low risk or high diversification approach. Different candidate portfolio weighting schemes only differ by the investor's choice of λ as well as the covariance matrix estimate Σ. The question is whether weighting schemes and covariance specifications exist which retain the desirable characteristics of market weights but without suffering from its drawbacks and which generate higher risk-adjusted returns.

3 Diversification

To construct a more diversified portfolio one could follow Choueifaty and Coignard (2008) who estimate a full covariance matrix Σ from past stock return data and then tailor their objective function to maximize diversification. While this approach is appealing in principle, optimization procedures that use estimates of a stock level covariance matrix on a large universe of stocks may result in concentrated, high-turnover portfolios unless they are tightly constrained (see, for example, Michaud, 1989). In fact, paradoxically, portfolios that are designed to be optimally diversified from a historical risk point of view often have highly concentrated holdings. Imposing tight and often ad-hoc constraints as a remedy, however, may defeat the purpose of a portfolio optimization. In general, it is well known that estimating a full covariance matrix robustly on a stock level can be challenging and always involves estimation error. In fact, DeMiguel *et al.* (2009) find that of 14 sample-based mean–variance models that they tested on different out-of-sample datasets none improved upon a "naïve" equal-weighted ($1/N$) strategy in terms of Sharpe ratio and other performance metrics.[8]

In order to mitigate these issues we propose a simple and transparent portfolio construction approach that requires no estimation and which results in a highly diversified portfolio, both on an individual stock and on a sector level. Our approach also generates portfolios with higher returns than MW indices at lower risk levels and with similarly low turnover and trading costs. The resulting diversified portfolio is a blend of MW and EW portfolios on a stock level and on a sector level.

To shrink market weights towards equal stock and sector weights combined, we decompose the covariance matrix Σ from Equation (1) and impose some structure on it.[9] We are then able to choose λ in a way that produces the desired blend of a highly diversified portfolio (equal stock and sector weights) and a highly scalable portfolio (market weights).

To illustrate our methodology, consider a simple risk model where each stock belongs to one sector only and sector risk is the only source of systematic (or common) risk (V). All remaining risk is company specific (Ω). As a consequence stocks in different sectors are uncorrelated and sectors are uncorrelated with each other. We split the covariance matrix Σ into a component for common sector risk and another for specific risk

$$\Sigma = a\Omega + bV$$

where a and b are scalars representing weights associated with residual and common risk respectively. We assume that there are N stocks in our universe and K sectors. If S is an $N \times K$ matrix of sector exposures where $s_{i,j}$ is 1 if stock i is a member of sector j and zero otherwise, then V can be expressed as

$$V = \mathrm{SFS}'$$

where F is a $K \times K$ diagonal sector covariance matrix. If we also assume that all sectors have the same risk and we standardize so that the risk of each sector is 1, then we can simplify this to

$$V = \mathrm{SS}'$$

where SS' is an $N \times N$ matrix of common sector exposures where element i, j of SS' is 1 if stocks

i and j are both in the same sector and zero otherwise.

In addition, we assume that specific risk is the same across stocks and standardize it. Hence Ω becomes a diagonal matrix of ones (an identity matrix **I**) and the covariance matrix can now be expressed as

$$\Sigma = \mathrm{aI} + \mathrm{bSS}'$$

So instead of Equation (1), we now have

$$w = (\lambda(\mathrm{aI} + \mathrm{bSS}') + (1-\lambda)M^{-1})^{-1}\iota \quad (2)$$

Equation (2) is a blend of market weights and equal stock and sector weights. If b is set to zero, we blend market weights with equal stock weights only. If a is set to zero, we blend market weights with equal sector weights only. The choice of λ depends on an investor's trade-off between his aversion to risk vs his aversion to trading costs.

Depending on the choice of parameters a, b and λ in Equation (2) we are able to combine market weights which are very efficient in terms of trading costs and turnover but tend to be concentrated on a stock and sector level with a weighting scheme that is highly diversified across stocks and sectors but which leads to higher turnover and limited capacity. A blend of these two weighting schemes allows us to achieve a highly effective combination as we demonstrate in the empirical analysis below.

An EW portfolio is the least concentrated or most highly diversified portfolio according to commonly used measures of concentration such as the Herfindahl Index or, more intuitively, its inverse which gives the effective number of stocks in a portfolio. For example, at the end of March 2013 the Herfindahl Index for the top 500 stocks in the US was 0.0069. Its inverse is 145 which suggests that even ignoring correlations across stocks, stock-level diversification in an MW portfolio of the top 500 stocks is much less than the 500 achievable in an EW strategy. As we discussed above, one appealing feature of an EW approach is that it mitigates the link between portfolio weights and market weights and therefore diminishes the negative relationship between market weights and subsequent stock returns. As a result, returns are enhanced.

Pure EW suffers from a range of drawbacks however. An EW portfolio is not easily implementable particularly if the universe includes illiquid small cap stocks since these would have very high portfolio weights relative to their market weights or "weight ratios". Moreover, maintaining equal weights over time requires high turnover. Therefore, we need to balance diversification on the one hand and implementability, capacity and trading costs on the other hand.

While diversification is most commonly analyzed in the context of risk reduction, an additional benefit of diversification is the return premium it generates. Following the previous literature, we refer to this return premium as the diversification return, which we investigate in more detail below.

3.1 The diversification return

The diversification return was first documented by Booth and Fama (1992). The authors demonstrate that a portfolio's compound return is greater than the weighted average of the compound returns of the assets in the portfolio. The incremental return is due to diversification. While Booth and Fama focused on diversification across asset classes, other authors have examined diversification returns within asset classes, e.g. Fernholz (2002), Choueifaty and Coignard (2008), Willenbrock (2011), Bouchey *et al.* (2012) or Plyakha *et al.* (2012).

Willenbrock (2011) shows that if we define g_p and g_i as the geometric average return of the portfolio and of stock i respectively and $w_{P,i}$ as the portfolio weight of stock i, then the following relationship holds

$$g_P \approx \sum w_{P,i} g_i + \underbrace{\frac{1}{2} \sum w_{P,i}(\sigma_i^2 - \sigma_P^2)}_{\text{Diversification return}} \quad (3)$$

where σ_i^2 and σ_P^2 are the stock and portfolio variance, respectively. Hence the second term on the right-hand side of Equation (3) is the portfolio's diversification return (DR_P). The DR_P term arises because once we have at least two assets in a portfolio and those assets are not perfectly correlated, then the portfolio's performance benefits from the lack of correlation of the returns of the assets. In this case the diversification return term is always positive as the portfolio variance is always lower than the weighted average of the individual asset variances. The more diversified a portfolio is, the higher its diversification return can be expected to be.

3.2 Capturing the diversification return

Only portfolios that rebalance regularly and thereby remain diversified are able to capture a diversification return. The magnitude of DR_P increases the more highly diversified a portfolio is. Also, the more frequently a portfolio is rebalanced, the more of its potential DR_p it will capture.

We derive in Appendix B an expression that links the returns of two idealized portfolios—an EW portfolio and an MW portfolio of the same assets. The return difference between these two portfolios is

$$\begin{aligned} r_{\mathrm{EW}} - r_{\mathrm{MW}} &= -N \cdot \mathrm{cov}(w_{\mathrm{MW},i}, r_i) \\ &\approx -\frac{1}{2} \mathrm{d} H_{\mathrm{MW}} + \mathrm{DR}_{\mathrm{EW}} \end{aligned} \quad (4)$$

The expression $-N \,\mathrm{cov}(w_{\mathrm{MW},i}, r_i)$ links market weights and returns. If market weights are positively (negatively) correlated with subsequent returns over some period, then an MW portfolio will outperform (underperform) an EW portfolio. If there is no relationship between market weights and returns, then there should be no difference in return between an MW portfolio and an EW portfolio. As Arnott *et al.* (2013) show, much of the returns of alternative indexing strategies can be traced to this simple relationship. Since alternative indexing strategies generally weaken the relationship between portfolio weights and market weights, their returns should be closer to an EW portfolio than an MW portfolio.

However, the second expression in Equation (4) is more instructive. It says that the return difference between an EW portfolio and an MW portfolio has two components—a change in MW portfolio concentration (measured by the change in the log of the Herfindahl Index of the MW portfolio) and the diversification return of the EW portfolio. As explained in Appendix B, the $\mathrm{DR}_{\mathrm{EW}}$ term arises because of the dispersion of returns which is *always positive*. However, if big stocks get bigger still (i.e. there is a positive relationship between market weights and subsequent returns) then the concentration of the MW portfolio increases. This component will negatively impact the return of an EW portfolio compared to an MW portfolio. As we will see, in some periods, this effect can dominate $\mathrm{DR}_{\mathrm{EW}}$ and cause an EW portfolio to underperform an MW portfolio.

However, the market cannot concentrate forever and therefore the long-run expected value of $\mathrm{d}H_{\mathrm{MW}}$ should be around zero. From Equation (4) this then implies that the expected excess return of an EW portfolio over an MW portfolio is $\mathrm{DR}_{\mathrm{EW}}$, i.e. the diversification return of the EW portfolio. Therefore, there is a strong case that more diversified strategies will outperform MW portfolios in the long run.

4 Data and methodology

We use market data from Factset and Global Industry Classification Standard (GICS) sector classifications for the empirical analysis. We derive our universe from the S&P Global Broad Market Index (BMI) constituents in the three main developed market regions, namely the United States, Japan and Europe.[10] We restrict this universe to the top 90% of the aggregate free float adjusted market value in each region (aggregated by country) in order to exclude potentially illiquid small and micro-cap stocks. The sample period used is from January 1992 through the end of March 2013.

We construct our diversified portfolios based on Equation (2) and compare them with an MW index. Portfolios are rebalanced annually at the end of December.[11] We report total returns (with dividends reinvested) before trading costs in GBP. We use the broadest GICS classification level which contains ten sectors in each region.

For the empirical analysis we construct portfolios based on a range of parameters a, b and λ from Equation (2) to demonstrate the robustness of our approach. As illustrated in Equation (2), these three parameters determine the amount of shrinkage towards equal stock and sector weights. We choose those parameters in a way to achieve certain levels of total portfolio shrinkage in percentage terms (TSP) and certain percentage contributions from stock and sector shrinkage (PCS_{Stock} and PCS_{Sec} respectively) to the total shrinkage as outlined in Appendix C.[12]

5 Empirical results

5.1 Performance and portfolio characteristics

Exhibit 2 examines performance and portfolio characteristics for the following parameter combinations:

TSP = 0.00, 0.20, 0.40, 0.60, 0.80 and 1.00

PCS_{Stock} (PCS_{Sec}) = 0.00 (1.00) ("Pure Sector Shrinkage"), 0.50 (0.50) ("50/50 Blend") and 1.00 (0.00) ("Pure Stock Shrinkage")

where $PCS_{Stock} + PCS_{Sec} = 1$.

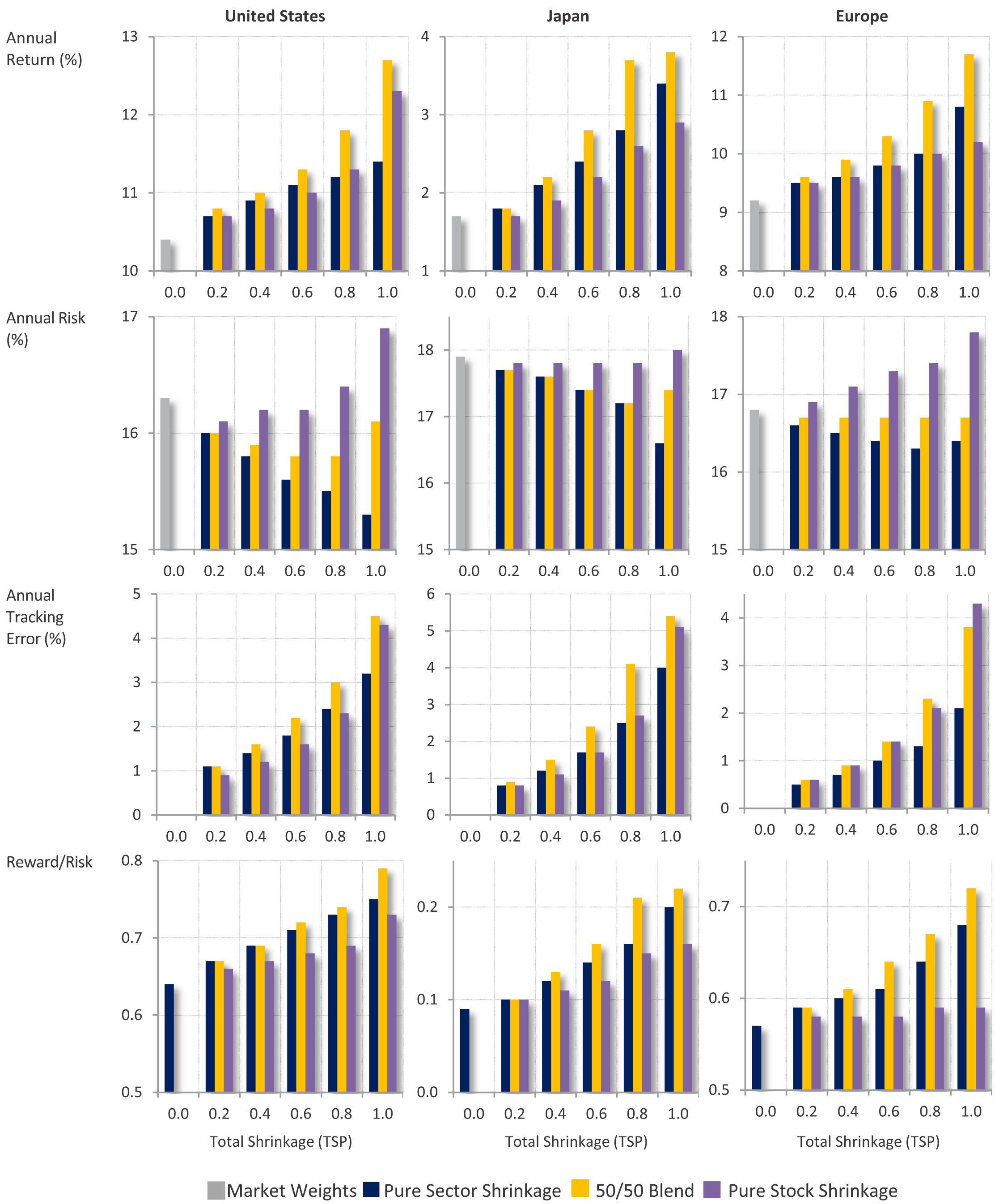
Different total shrinkage levels (TSP) are shown on the horizontal axis of each chart and different stock and sector percentage contributions (PCS_{Stock} and PCS_{Sec} respectively) are shown in different colors. The grey bars show statistics for an MW portfolio with no shrinkage applied. The dark blue bars in each chart (labeled "Pure Sector Shrinkage") target pure sector level shrinkage and no stock level shrinkage. So, for example, in the US, the annual return increases from 10.4% to 11.4% as we shrink to equal sector weights in TSP increments of 0.2. Any stock level shrinkage that arises is purely a result of the sector shrinkage. The purple bars in each chart (labeled "Pure Stock Shrinkage") target pure stock level shrinkage and no sector level shrinkage. Again, any sector shrinkage that arises is purely a result of the stock level shrinkage. The orange bars ("50/50 Blend") show a blend of equal stock and sector level shrinkage contributions. The last three bars in each chart (for TSP = 1.0) show the results for a fully equal stock-weighted portfolio ("Pure Stock Shrinkage"), a fully equal sector weighted portfolio ("Pure Sector Shrinkage") and for the 50/50 blend with the maximum shrinkage (TSP) possible.[13]

Panel A of Exhibit 2 shows performance, risk and turnover information for the different parameter combinations we test. Over the sample period from 1992 to 2013 all our blends outperform market weights. The first set of charts show that combining stock and sector shrinkage almost always generates higher returns than either of the two in isolation for any given level of TSP. This type of strategy outperforms an MW index by up to 250 basis points per year depending on how far we are willing to deviate from market weights. In general, returns increase approximately linearly the more we diversify compared to market weights.

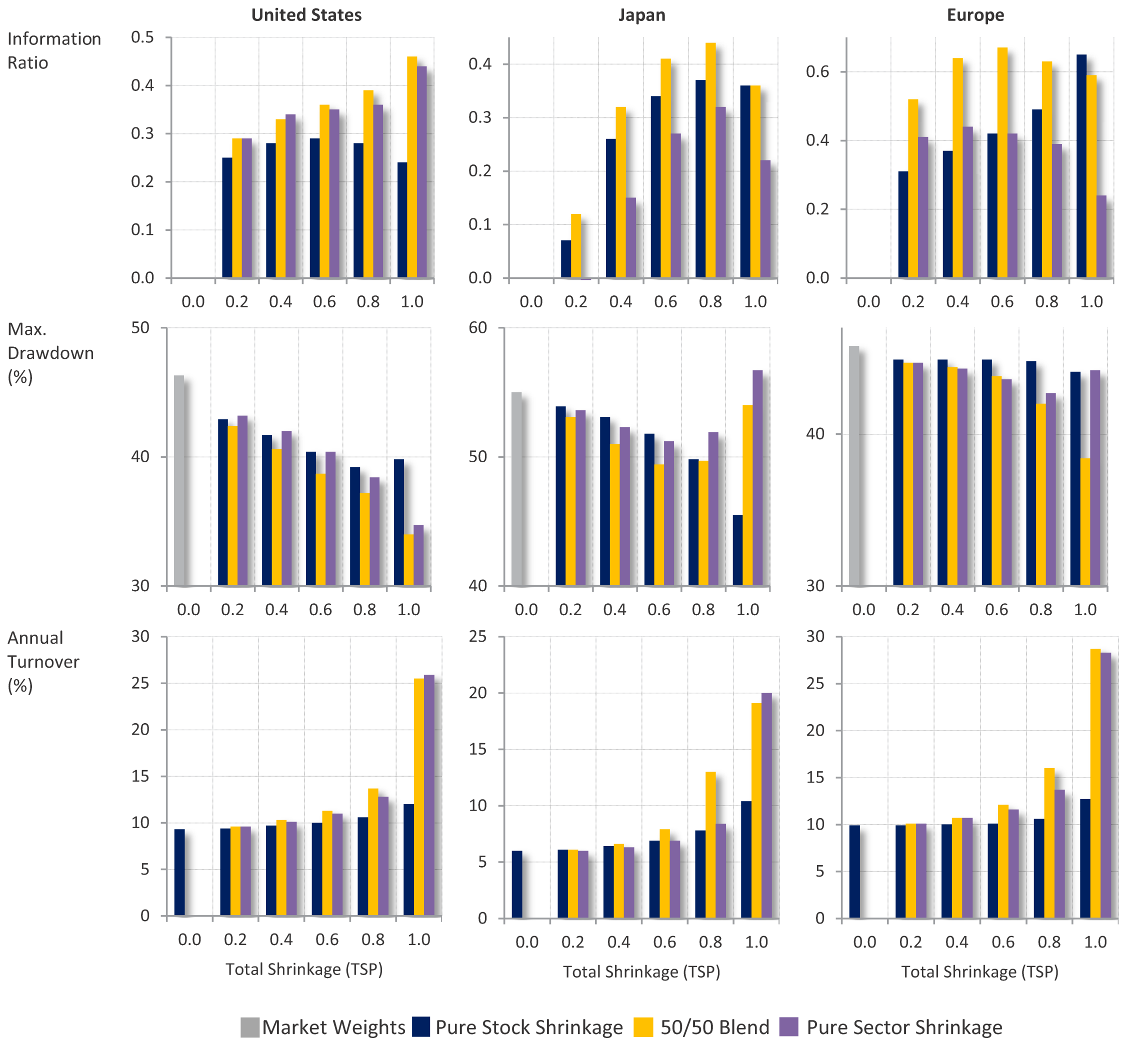
Portfolio risk (shown in the second set of charts in Panel A) for pure stock shrinkage is generally

Exhibit 2 Diversified portfolio characteristics (1992–2013).
Panel A: Portfolio performance, risk and turnover

Different total shrinkage levels (TSP) are shown on the horizontal axis of each chart and different stock and sector percentage contributions (PCS_{Stock} and PCS_{Sec} respectively) are shown in different colors. The grey bars show statistics for an MW portfolio with no shrinkage applied. The dark blue bars in each chart (labeled "Pure Sector Shrinkage") target pure sector level shrinkage and no stock level shrinkage. The purple bars in each chart (labeled "Pure Stock Shrinkage") target pure stock level shrinkage and no sector level shrinkage. The orange bars ("50/50 Blend") show a blend of equal stock and sector level shrinkage contributions. The last two bars in each chart (for TSP = 1.0) show the results for a fully equal stock-weighted portfolio ("Pure Stock Shrinkage") and a fully equal sector-weighted portfolio ("Pure Sector Shrinkage").



Panel A (*Continued*)



Source: Global Systematic Investors, based on Factset/Standard and Poors data.

similar to an MW portfolio's risk or higher and as we shrink all the way to equal stock weights, risk is always higher than for an MW portfolio. This pattern may seem surprising as we create more diversified portfolios which should therefore have lower risk. However, we also increase exposure to smaller stocks which tend to be more risky than larger ones. Moreover, purely shrinking towards equal stock weights may inhibit diversification on a sector level as the resulting sector weights are simply proportional to the number of stocks in an EW portfolio. In contrast, sector shrinkage exhibits consistently lower risk levels than an MW portfolio as does combined stock and sector shrinkage. For a given level of TSP pure sector shrinkage generally results in the lowest risk portfolio across all combinations.

Our diversification strategy does not explicitly take tracking error into account as we do not anchor on an MW portfolio but instead aim to provide a superior alternative. Nevertheless, it is instructive to observe the resulting portfolio tracking error levels relative to an MW index (shown in the third set of graphs in Panel A). Tracking errors range between 0.5% and 5.5% depending how far we deviate from the MW index. Mostly tracking errors are within the 1.5–4% range. Hence active risk levels are relatively modest which emphasizes the fact that our diversified portfolios are not extreme according to standard measures.

Reward-to-risk ratios (shown in the fourth set of charts in Panel A) substantially exceed those of an MW portfolio for all parameter combinations tested. The consistency of this finding over a range of different TSP and PCS_{Stock} shrinkage levels reinforces its robustness. For a given TSP level, we can observe that pure sector shrinkage always outperforms pure stock shrinkage in terms of risk-adjusted performance. However, our blend almost always outperforms both of these more "extreme" options for any given level of TSP while resulting in more balanced and more implementable portfolios as we will demonstrate below.

Information ratios for our diversified portfolios are shown in the fifth set of graphs in Panel A. They range from approximately 0 to 0.65. For a given TSP level information ratios are almost always highest for our blend which combines equal stock and equal sector shrinkage. However, we should point out that our objective in this study is not to maximize information ratios but to maximize reward-to-risk ratios while retaining sizable portfolio capacity and scalability.

Maximum drawdowns (shown in the sixth set of charts in Panel A) over the entire sample period are almost always highest for the MW portfolio. Depending on the region they can sometimes be lower for stock shrinkage and sometimes for sector shrinkage, thereby providing further rationale for combining stock and sector shrinkage which tends to result in the lowest maximum drawdowns for any given TSP level.

Annual turnover (shown in the last set of charts in Panel A) is only slightly higher than for an MW index, especially for lower TSP levels. In particular, sector shrinkage exhibits only marginally higher turnover levels than an MW index. Stock and sector shrinkage combined leads to turnover levels that are at most twice the MW turnover. As expected, shrinking all the way to equal stock weights leads to considerably higher turnover levels and is therefore more challenging and costly to implement.

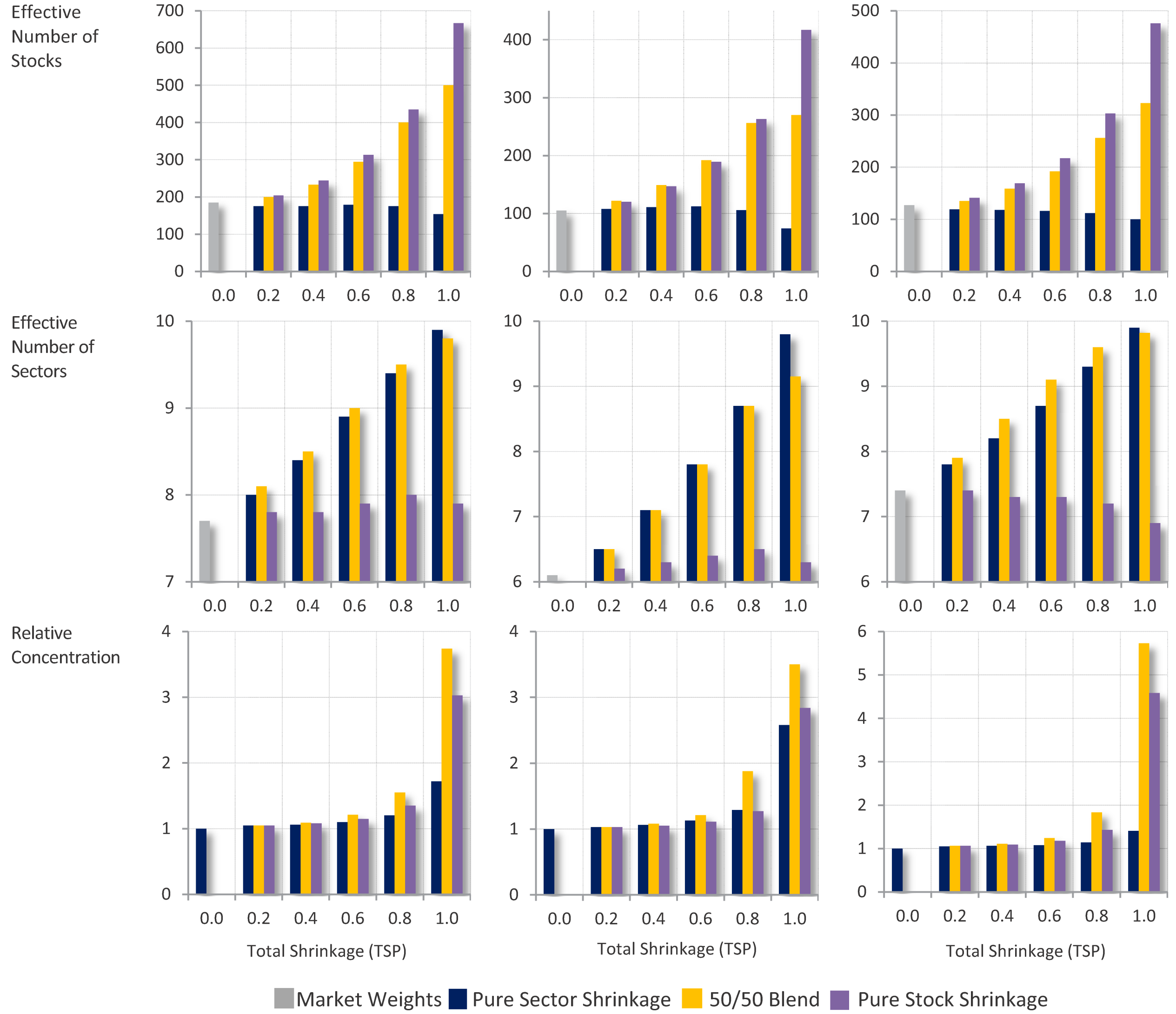
Panel B of Exhibit 2 shows absolute and relative portfolio concentration measures averaged over our sample period. The effective number of stocks and sectors (shown in the first and the second set of charts in Panel B) are computed as the inverse of the Herfindahl Index on a stock and sector level, respectively. Shrinking at the stock or sector level in isolation leads to a higher effective number of stocks or sectors, respectively. However, increasing only sector-level diversification ("Pure Sector Shrinkage") in fact tends to result in a slight decrease in the effective number of stocks and hence a decrease in stock-level diversification. This scenario arises when we have large companies in a small sector as, for example, in the Telecommunications sector. Similarly, increasing only stock-level diversification ("Pure Stock Shrinkage") can result in a decrease in the effective number of sectors and hence a decrease in sector-level diversification. This may contribute to the increase in risk observed at high levels of stock shrinkage seen in Panel A. Only by combining stock and sector shrinkage do we ensure that portfolios are well diversified on both levels.

The last set of charts in Panel B show the Relative Concentration Index (RCI) of each portfolio which we compute as the portfolio-weighted

Exhibit 2

Panel B: Portfolio concentration

Different total shrinkage levels (TSP) are shown on the horizontal axis of each chart and different stock and sector percentage contributions (PCS_{Stock} and PCS_{Sec}, respectively) are shown in different colors. The grey bars show statistics for an MW portfolio with no shrinkage applied. The dark blue bars in each chart (labeled "Pure Sector Shrinkage") target pure sector level shrinkage and no stock level shrinkage. The purple bars in each chart (labeled "Pure Stock Shrinkage") target pure stock level shrinkage and no sector level shrinkage. The orange bars ("50/50 Blend") show a blend of equal stock and sector level shrinkage contributions. The last two bars in each chart (for TSP = 1.0) show the results for a fully equal stock-weighted portfolio ("Pure Stock Shrinkage") and a fully equal sector-weighted portfolio ("Pure Sector Shrinkage").



Source: Global Systematic Investors, based on Factset/Standard and Poors data.

average of its stock weights relative to market weights. Relative concentration is a measure of investment capacity and portfolio scalability. An MW portfolio has the highest achievable investment capacity and therefore the lowest relative concentration (RCI = 1). A higher relative concentration level (for example RCI = 2) means that a portfolio has lower investment capacity than market weights (in this case half the investment capacity). Exhibit 2 shows that the investment capacity of our diversified portfolios is generally very high. The relative concentration index is mostly well below two and only increases more strongly once we move all the way to equal stock or sector weights.

Hence, thus far we can conclude the following: (1) Both stock and sector shrinkage deliver better performance than market weights. However, combining the two components maximizes the benefit and consistently generates higher returns than market weights at lower risk. (2) Reward-to-risk ratios generally increase the further we diversify relative to market weights while turnover only increases marginally unless we move all the way to equal stock weights. (3) Diversification is increased both on a stock level and on a sector level. (4) The investment capacity of the strategy is extremely high, at least half the capacity of MW indices.

As a result, our simple blended approach between market weights and equal stock and sector weights appears to retain the desirable characteristics of a truly passive portfolio but improves performance. The enhanced performance results from better diversification and its associated higher diversification return. The resulting portfolios have low turnover, high capacity and liquidity and generate an annualized outperformance of up to 250 basis points from 1992–2013.

5.2 *Integrated model—relative return decomposition*

As we have seen above, an integrated model which shrinks market weights towards both equal stock and sector weights delivers the strongest performance. In order to investigate the sources of outperformance of this type of diversified portfolio over market weights in more detail, we focus on a portfolio that targets equal percentage contributions to total shrinkage from a stock level and from a sector level ($\mathrm{PCS}_{\mathrm{Stock}} = \mathrm{PCS}_{\mathrm{Sec}} = 0.50$). We examine two different total shrinkage levels for this portfolio, namely $\mathrm{TSP} = 0.60$ and $\mathrm{TSP} = 0.80$. Which level of TSP to choose ultimately depends on an investor's preferences. In the following we call the resulting portfolios "the diversified portfolios".

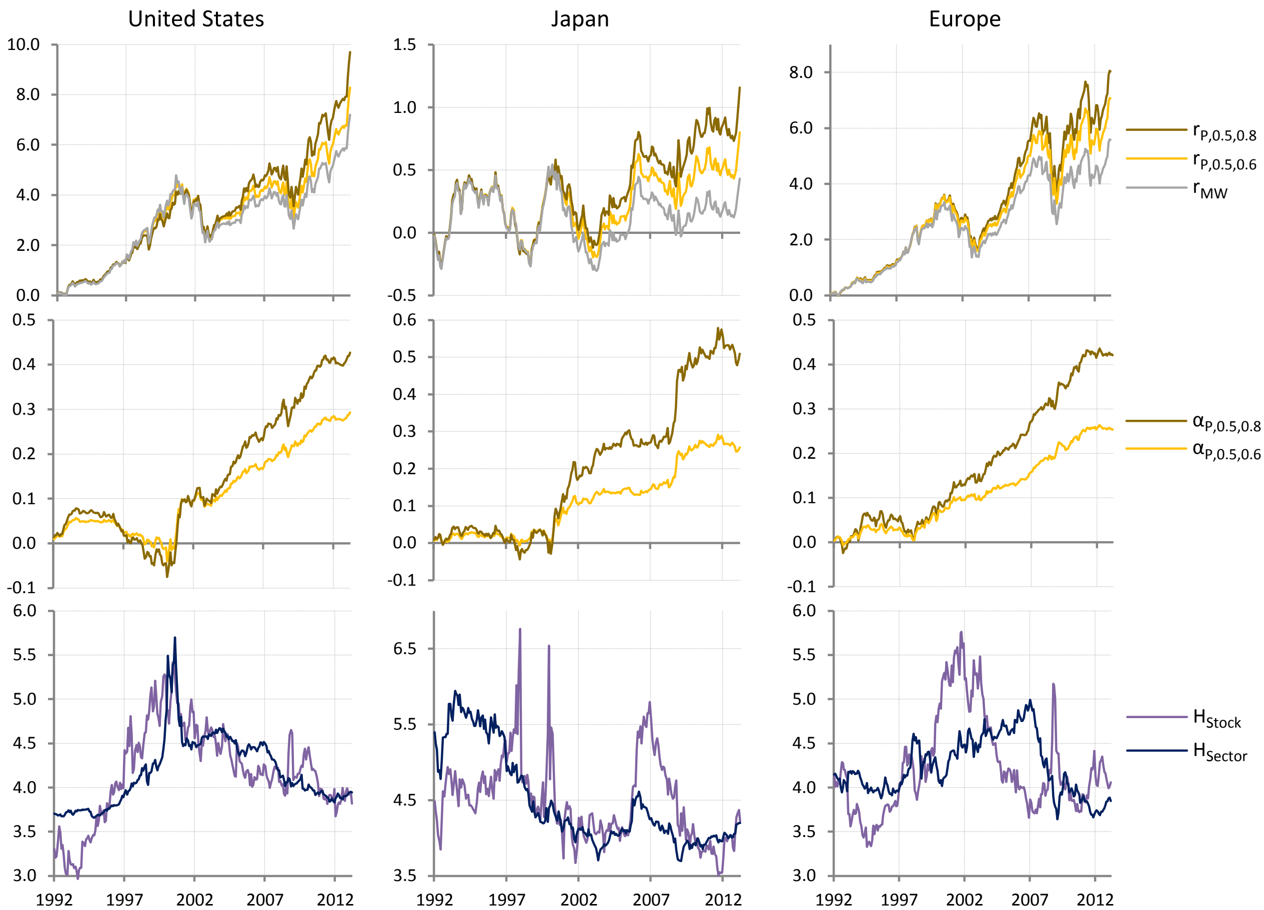
Exhibit 3 compares diversified portfolio returns with MW returns and illustrates how relative returns co-move with market concentration levels. The first set of graphs in Exhibit 3 shows cumulative returns of each of the diversified portfolios and the MW portfolio (denoted $r_{P,0.5,0.6}$, $r_{P,0.5,0.8}$ and r_{MW}, respectively). The second set of graphs shows cumulative residual returns resulting from a time series regression of diversified portfolio returns regressed on an intercept and on market returns (denoted $\alpha_{P,0.5,0.6}$ and $\alpha_{P,0.5,0.8}$, respectively).[13] As a result, the residual returns are adjusted for the market risk of each portfolio. We can observe that the diversified portfolios consistently outperformed the MW portfolio with the exception of the tech boom around the end of the 1990s, particularly in the US.

The third set of graphs in Exhibit 3 shows the MW portfolio's Herfindahl Index, i.e. its concentration level, both on a stock level (H_{MW}) and on a sector level ($H_{\mathrm{MW,Sec}}$).[14]

We can observe that during the technology boom around the end of the 1990s market concentration on a stock level increased substantially in the US and in Europe. In the US, sector level concentration increased strongly at the same time as the technology sector boomed. During this period the increase in market concentration dominated the diversification return. Hence the diversified portfolios underperformed the MW portfolio. Once

Exhibit 3 Diversified portfolios vs market weighted index—Relative return decomposition (1992–2013).

The first set of graphs shows cumulative returns of each of the diversified portfolios and the MW portfolio (denoted $r_{P,0.5,0.6}$, $r_{P,0.5,0.8}$ and r_{MW}, respectively). The second set of graphs shows cumulative residual returns resulting from a time series regression of diversified portfolio returns regressed on an intercept and on market returns (denoted $\alpha_{P,0.5,0.6}$ and $\alpha_{P,0.5,0.8}$, respectively). The third set of graphs shows the MW portfolio's Herfindahl Index, i.e. its concentration level, both on a stock level (H_{MW}) and on a sector level ($H_{\mathrm{MW,Sec}}$).



Source: Global Systematic Investors, based on Factset/Standard and Poors data.

the tech boom burst in 2000, market concentration levels fell again. As a result the diversified portfolios were able to capture their diversification returns and generated positive relative returns.

It is also interesting to observe that while sometimes stock and sector concentration levels move together (such as during the tech boom in the US), they move in opposite directions at other times. This provides further justification for combining individual stock shrinkage with sector shrinkage as the two components have the potential to diversify each other, thereby generating a smoother relative return series.

In order to test our (approximate) decomposition of relative returns from Equation (4) more formally, we also run the following time series regression models[15]:

Model 1: $r_P = \alpha + \beta r_M + \varepsilon$
Model 2: Model $1 + \gamma \mathrm{d} H_{\mathrm{MW}}$
Model 3: Model $1 + \delta[-0.5\mathrm{d} H_{\mathrm{MW}} + \mathrm{DR}_{\mathrm{EW}}]$

Model 4: Model 1 + $\delta_{\text{Sec}}[-0.5\mathrm{d}H_{\text{MW,Sec}} + \text{DR}_{\text{EW,Sec}}]$
Model 5: Model 1 + $\theta_{\text{SMB}}\text{SMB} + \theta_{\text{HML}}\text{HML} + \theta_{\text{MOM}}\text{MOM}$
Model 6: Model 3 + Model 5

Model 1 is a market model where we regress diversified portfolio returns on market returns only. Model 2 adds the change in the MW portfolio concentration term ($\mathrm{d}H_{\text{MW}}$) from Equation (4) to Model 1. Model 3 adds the entire decomposition term from Equation (4) to Model 1. Model 4 adds the same term measured on a sector level to Model 1. Model 5 is simply the well-known Fama–French–Carhart four-factor model (the market, SMB, HML and momentum (MOM)) for comparison.[16] Model 6 combines Models 3 and 5 in order to examine whether our relative return components remain statistically significant over and above the widely used four-factor model.

We measure the diversification return (DR_P from Equation (3)) at time t as follows:

$$\begin{aligned}\text{DR}_P &= \frac{1}{2}\sum w_{P,i}(\sigma_i^2 - \sigma_P^2)\\ &\approx \frac{1}{2}\sum w_{P,i}(r_i^2 - r_P^2)\end{aligned}$$

Hence we replace the more conventional time series estimates of σ_i^2 and σ_P^2 with their cross-sectional estimates which are more timely measures of risk that change every period (as shown by Yu and Sharaiha, 2007) as well as several other studies). This relationship also demonstrates that the diversification return (DR_P) is equal to approximately half the cross-sectional return dispersion of the stocks included in a portfolio.

The regression results are shown in Exhibit 4 for our three regions. For each regression model and each regression coefficient estimate we show the results for each of our two diversified portfolios which only differ by TSP (TSP = 0.6 is shown first and then TSP = 0.8). T-statistics are shown in parentheses. The market model results (Model 1) indicate that in each region the diversified portfolios generate statistically significant outperformance after controlling for market movements. The intercept coefficients α are all positive and almost all are highly statistically significant. At the same time the diversified portfolios exhibit market betas that are significantly below one in each region indicating that their market risk is lower than for an MW portfolio. Annualizing the α coefficient estimates leads to a market risk-adjusted outperformance of approximately 100–200 basis points for all regions depending on the total shrinkage level (TSP) chosen.

Adding the change in market portfolio concentration term ($\mathrm{d}H_{\text{MW}}$) to Model 1 (Model 2) increases the α coefficients substantially. Depending on the region and on TSP the annualized α increase ranges between 35 and 135 basis points. As expected from Equation (4), the regression coefficients on the newly added term are always negative and highly statistically significant. In line with our illustration from above, the relative portfolio returns increase when we condition them on the change in market concentration. As expected, the loading on $\mathrm{d}H_{\text{MW}}$ always increases in absolute magnitude for higher TSP.

When we add our relative return decomposition term from Equation (4) (Model 3), this term can be seen to be positive and highly statistically significant in all regions as expected. Again, the regression coefficient increases in magnitude as we move to a higher level of TSP as expected. The diversified portfolios suffer in relative terms if the market becomes more concentrated as stocks with higher market weights exhibit relatively higher returns than those with lower weights. As the diversified portfolios are underweight the former

Exhibit 4 Diversified portfolio—relative return decomposition.

Model 1: $r_P = \alpha + \beta r_M + \varepsilon$

Model 2: Model 1 + $\gamma \mathrm{d}H_{\mathrm{MW}}$

Model 3: Model 1 + $\delta[-0.5\mathrm{d}H_{\mathrm{MW}} + \mathrm{DR}_{\mathrm{EW}}]$

Model 4: Model 1 + $\delta_{\mathrm{Sec}}[-0.5\mathrm{d}H_{\mathrm{MW,Sec}} + \mathrm{DR}_{\mathrm{EW,Sec}}]$

Model 5: Model 1 + $\theta_{\mathrm{SMB}}\mathrm{SMB} + \theta_{\mathrm{HML}}\mathrm{HML} + \theta_{\mathrm{MOM}}\mathrm{MOM}$

Model 6: Model 3 + Model 5

Monthly data: 01/1992–03/2013. T-statistics are shown in parentheses. T-statistics for β coefficients are shown for a null of $\beta = 1$. Results for the two diversified portfolios from above are shown ($\mathrm{PCS}_{\mathrm{Stock}} = \mathrm{PCS}_{\mathrm{Sec}} = 0.50$ and TSP = 0.60 (first value) and 0.80 (second value)).

		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
United States	α	0.0010 (2.7)	0.0017 (5.5)	0.0001 (0.4)	0.0001 (0.6)	0.0009 (2.7)	−0.0002 (−0.7)
		0.0014 (2.6)	0.0025 (6.5)	−0.0001 (−0.2)	0.0003 (0.8)	0.0013 (2.7)	−0.0004 (−1.1)
	β	0.9596 (−5.1)	0.9505 (−7.9)	0.9494 (−7.8)	0.9933 (−1.4)	0.9534 (−6.3)	0.9581 (−7.4)
		0.9527 (−4.2)	0.9377 (−7.9)	0.9356 (−7.9)	0.9956 (−0.5)	0.9369 (−5.7)	0.9447 (−7.2)
	γ		−0.1716 (−12.7)				
			−0.2806 (−16.6)				
	δ			0.3224 (11.7)			0.3349 (13.5)
				0.5373 (15.5)			0.5504 (16.3)
	δ_{Sec}				0.5998 (22.8)		
					0.7642 (17.4)		
	θ_{SMB}					−0.0081 (−1.0)	−0.0448 (−6.5)
						0.0131 (1.1)	−0.0471 (−5.0)
	θ_{HML}					0.0786 (10.0)	0.0616 (10.0)
						0.0939 (8.0)	0.0659 (7.9)
	θ_{MOM}					−0.0191 (−3.2)	0.0043 (0.9)
						−0.0332 (−3.8)	0.0053 (0.8)
	Adj.R^2	98.3%	98.9%	98.9%	99.4%	98.8%	99.3%
		96.5%	98.3%	98.2%	98.4%	97.3%	98.7%

Exhibit 4 (*Continued*)

		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Japan							
	α	0.0009 (2.2)	0.0012 (4.6)	0.0002 (0.8)	0.0001 (0.2)	0.0006 (1.6)	−0.0001 (−0.3)
		0.0017 (2.4)	0.0021 (4.7)	0.0004 (1.0)	0.0003 (0.6)	0.0012 (2.0)	0.0002 (0.5)
	β	0.9633 (−4.5)	0.9616 (−7.6)	0.9586 (−8.6)	0.9806 (−3.2)	0.9583 (−5.9)	0.9600 (−8.9)
		0.933 (−4.9)	0.9302 (−8.0)	0.9250 (−8.9)	0.9600 (−3.6)	0.9202 (−6.8)	0.9227 (−9.8)
	γ		−0.1418 (−19.8)				
			−0.2386 (19.3)				
	δ			0.2935 (21.4)			0.2885 (19.5)
				0.4939 (20.8)			0.4589 (17.7)
	δ_{Sec}				0.6569 (14.6)		
					1.0226 (12.5)		
	θ_{SMB}					0.0348 (3.6)	−0.0153 (−2.3)
						0.0933 (5.8)	0.0136 (1.2)
	θ_{HML}					0.0663 (6.3)	0.0420 (6.2)
						0.0911 (5.2)	0.0524 (4.4)
	θ_{MOM}					−0.0234 (−3.5)	0.0116 (2.5)
						−0.0398 (−3.6)	0.0158 (2.0)
	Adj.R^2	98.2%	99.3%	99.4%	99.0%	98.7%	99.5%
		94.8%	97.9%	98.1%	96.8%	96.3%	98.3%

Exhibit 4 (*Continued*)

		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Europe							
	α	0.0009 (3.5)	0.0013 (5.6)	0.0006 (2.7)	0.0005 (2.1)	0.0009 (3.7)	0.0004 (1.6)
		0.0014 (3.3)	0.0021 (5.8)	0.0009 (2.4)	0.0009 (2.1)	0.0016 (4.0)	0.0007 (1.9)
	β	0.9866 (−2.5)	0.9636 (−6.8)	0.9633 (−6.8)	1.0054 (1.0)	0.9908 (−1.6)	0.9675 (−5.9)
		0.9796 (−2.3)	0.9365 (−7.4)	0.9365 (−7.3)	1.0041 (0.4)	0.9852 (−1.6)	0.9474 (−5.9)
	γ		−0.0892 (−8.9)				
			−0.1672 (−10.4)				
	δ			0.1755 (8.8)			0.2027 (8.9)
				0.3247 (10.1)			0.3291 (9.0)
	δ_{Sec}				0.2736 (8.1)		
					0.3561 (6.0)		
	θ_{SMB}					0.0397 (4.2)	0.0095 (1.1)
						0.0902 (6.0)	0.0413 (2.9)
	θ_{HML}					−0.0101 (−1.2)	−0.0143 (−2.0)
						−0.0171 (−1.3)	−0.0241 (−2.1)
	θ_{MOM}					−0.0043 (−0.8)	0.0192 (3.6)
						−0.0223 (−2.5)	0.0159 (1.8)
	Adj.R^2	99.3%	99.4%	99.4%	99.4%	99.3%	99.5%
		97.9%	98.6%	98.5%	98.2%	98.2%	98.6%

Source: Global Systematic Investors, based on Factset/Standard and Poors data.

and overweight the latter relative to the MW portfolio, their relative returns are negatively associated with changes in market concentration. However, changing market concentration levels are more than offset over time by the diversification return which is always positive. Moreover, we can observe that once our $-0.5\mathrm{d}H_{\mathrm{MW}} + \mathrm{DR}_{\mathrm{EW}}$ term is included in the model, the regression intercept α loses its statistical significance in the United States and Japan although it remains positive and significant in Europe.

Model 4 adds our decomposition term measured on a sector level instead of a stock level to Model 1. Just as for the stock level term in Model 3 we can observe that the sector level term is highly statistically significant and increases with TSP. Interestingly, the market beta of the portfolio increases substantially in all regions if we include a sector level term instead of a stock-level term.

Model 5 shows a standard four-factor model (the market, size (SMB), value (HML) and momentum (MOM)). It is interesting to observe that all regression intercepts (α) remain positive and statistically significant in all three regions (except for Japan where α is positive but not statistically significant at lower TSP levels). This indicates that our simple diversification procedure tends to generate significantly positive returns even after controlling for a four-factor model. Moreover, we can observe that the adjusted R^2 of Model 4 is almost always higher than for Model 5.

Model 6 combines Models 2 and 5. We can observe that once we add the four-factor model to Model 2 our relative return decomposition term remains highly statistically significant. In fact, its statistical significance remains almost unchanged. In other words, it is not subsumed by a standard four-factor model. One puzzling finding is that the regression intercept (α) in Europe remains highly statistically significant across almost all model versions tested. A possible reason might be that we are inadvertently exploiting a country or currency effect which is not incorporated into our model. This is a possibility since Europe is the only multi-country and multi-currency region in our data sample.

5.3 *Diversified portfolio characteristics*

We have demonstrated that the diversified portfolios consistently outperform an MW index over our sample period. We have also been able to show that the diversified portfolios' relative returns can be decomposed into an MW return term as well as the change in MW portfolio concentration and the portfolios' diversification return.

To obtain a better understanding of the nature of the diversified portfolios that we propose and in order to evaluate their implementability and scalability in practice, we examine a snapshot of our two diversified portfolios at the end of our sample period (31 March 2013). The portfolio characteristics we examine are summarized in Exhibit 5.[17] We show average market values, accounting ratios as well as portfolio concentration proxies on a stock and sector level for each of the three regions we examine.

While average market caps are lower for the diversified portfolios than for the MW portfolio all averages are well within a large cap range as they vary between about £7 and £35 billion across regions and levels of TSP which compares to £17 and £48 billion for market weights. Book/market ratios as well as dividend yields of the diversified portfolios tend to be similar to the MW portfolio. The diversified portfolios do not exhibit any consistent value or growth tilt compared to market weights. Over our entire sample period we observe from Exhibit 4, however, that there is a significant loading on HML in the regressions in the US and Japan which indicates a value tilt. Earnings yields tend to be lower for

Exhibit 5 Diversified portfolio characteristics (as of 31 March 2013).

The table above shows portfolio characteristics of the diversified portfolios ($PCS_{Stock} = PCS_{Sec} = 0.50$ and $TSP = 0.60$ (first value) and 0.80 (second value)) and an MW portfolio for the three regions we examine.

	United States		Japan		Europe	
Characteristics	Diversified portfolio	Market weights	Diversified portfolio	Market weights	Diversified portfolio	Market weights
Number of securities	732	771	444	461	507	528
Weighted average Mkt Cap	35,264	48,439	11,090	17,329	27,922	38,204
(million £)	25,841		7,106		21,050	
Weighted average book/market	0.40	0.41	0.86	0.84	0.63	0.64
	0.41		0.90		0.62	
Weighted average dividend yield (%)	1.82	1.80	2.10	2.05	3.78	3.71
	1.80		2.11		3.78	
Weighted average earnings yield (%)	5.15	5.45	2.77	3.78	6.03	6.32
	4.84		1.72		5.49	
Aggregate Top-10 stock weight (%)	8.8	13.5	12.3	21.4	12.0	19.8
	6.2		10.3		8.6	
Aggregate Top-3 sector weight (%)	38.0	46.3	48.2	61.6	38.8	47.5
	34.7		40.5		35.3	

Source: Global Systematic Investors, based on Factset/Standard and Poors data.

the diversified portfolios than for the MW portfolio. Over our entire sample period (not shown), however, the diversified portfolio mitigates some of the well-known growth bias inherent to market weights, which benefits diversified portfolio returns to some degree.

By design the largest stock weights are likely to be lower for the diversified portfolios than for MW indices. This pattern is reflected in the aggregate top-10 stock weights shown and holds in all three regions. By construction, this pattern is strongest for higher TSP levels. For $TSP = 0.8$ aggregate top-10 portfolio stock weights are less than half the aggregate top-10 market weights in all three regions.

As we blend market weights with equal sector weights in addition to equal stock weights, our diversified portfolios exhibit more balanced sector allocations than market weights. Sectors with below-average market weights tend to have their weights increased and sectors with above-average market weights tend to be reduced, though not uniformly since we shrink towards equal stock weights at the same time. This pattern is reflected in the aggregate top-3 sector weights that we show below for the MW portfolio as well as the diversified portfolios. For example, for $TSP = 0.8$ the aggregate top-3 portfolio sector weights are only about three-quarters of the aggregate top-3 sector market weights.

We have seen in Exhibit 2 that the proposed diversified portfolio versions ($PCS_{Stock} = 0.50$ and $TSP = 0.60$ or 0.80) outperformed an MW index by approximately 90–200 basis points per annum while their annual turnover is only about 2–7 percentage points higher. A simple back-of-the envelope calculation demonstrates that the per-trade transaction cost (roughly speaking commission, bid–ask spread and market impact) would have to be unrealistically high in order to offset the return benefit of the diversified portfolio vs an MW index. It is important to remember that the diversified portfolio's average market cap, although lower than for an MW index, is mostly in the double-digit billions and we have excluded small cap stocks (the bottom 10% of the aggregate market capitalization) from our universe which constitute the least liquid market segment. As a result, we can safely dismiss the possibility that the return advantage will be eliminated by transaction costs or that it is attributable to a micro-cap effect.

5.4 Potential extensions

Our diversified portfolio can either be invested in directly or it can be used as a base portfolio for a factor overlay. In other words, one could overweight and underweight stocks relative to the diversified portfolio based on factors such as value, momentum and low risk. Using the diversified portfolio instead of an MW portfolio as a base portfolio delivers an expected return premium in addition to any potential factor premium. Moreover, it distorts factor exposures less than an MW portfolio and allows factor premia to be captured in a more balanced way than an MW portfolio due to its better diversification. Furthermore, factor effects are often weaker in large and mega-cap stocks and stronger in a mid and small cap universe. Anchoring factor bets on market weights is therefore likely to be suboptimal.

Another interesting area of research would be to link our diversified portfolio to mutual fund performance. Petajisto (2013) has shown that the most active stock pickers (those with the highest "active share") tend to have the best performance and that cross-sectional dispersion in stock returns positively predicts performance by stock pickers. As the diversification return discussed above can be measured using cross-sectional dispersion in stock returns, there is a direct link between this relative return component and the observed return of active stock pickers. The question arises how much of their realized relative returns can be explained (and therefore captured) by our diversified strategy in a scalable and low turnover manner and at a very low cost.

6 Conclusion

We have presented a simple portfolio construction approach which is a blend of market weights and equal stock and sector weights. Our approach results in a highly diversified portfolio both on a stock level and on a sector level and generates higher portfolio returns at lower risk than an MW index. The higher returns of our diversified portfolio originate both from mitigating the link with market weights and from its higher diversification return which we are able to capture because we rebalance our portfolio on a regular basis.

Our diversified portfolio exhibits only slightly higher turnover than an MW index and is less concentrated in mega-cap stocks. Instead, it assigns somewhat higher weight to smaller stocks and sectors than an MW index. At the same time the diversified portfolio retains the characteristics of a broad market index and is therefore highly implementable and has very high investment capacity.

We make the following contributions to the existing literature: (1) We combine stock and sector-level diversification by shrinking towards equal stock and sector weights simultaneously; (2) We decompose equal-weighted relative portfolio returns into a change in market concentration term and a diversification return term; and (3) We demonstrate that there is a straightforward portfolio construction methodology that allows us to exploit these findings in a scalable, low turnover and therefore easily implementable portfolio.

Our simple diversification strategy appears to be a superior way of capturing the equity risk premium compared to an MW portfolio. It therefore establishes a tougher benchmark for active fund managers than MW indices as it results in well-diversified, high-capacity and low turnover portfolios that can be delivered very cheaply.

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Appendices

A. Derivation of utility function

In its most general form, an investor's utility function is usually expressed as follows:

$$U(w) = w'r - \frac{1}{2}\lambda w'\Sigma w - \mathrm{TC}(w - w_0) \quad \text{(A1)}$$

where $\mathrm{TC}(w - w)$ represents trading costs as a function of the change in the weights from w_0, the starting asset weights, to w, the new optimal weights. This specification is an improvement on the trading cost free optimization but introduces the issue that trading costs will likely differ for different portfolios. Large portfolios require larger trades to rebalance and therefore tend to incur larger trading costs. Smaller portfolios may be affected by minimum trade sizes and ticket charges. Each portfolio will have a different set of starting weights w_0 and this will influence the amount of trading needed to rebalance each portfolio. The portfolio-specific nature of trading costs is undesirable for our purposes as we wish to model a single model or target portfolio for all portfolios following the same strategy.

A simple proxy for trading costs, particularly market impact costs, is to use the desired portfolio weight in a stock and divide it by its market weight.[18] This ratio corresponds to the *Tradesize* variable of Keim and Madhavan (1997) which has been shown to be one of the strongest predictors of trading costs.[19]

The statistic we focus on is therefore $w_{P,i}/w_{\mathrm{MW},i}$ where $w_{P,i}$ is the desired weight of stock i in the portfolio and $w_{\mathrm{MW},i}$ is the market weight of stock i. Summing these relative weights across the portfolio and weighting them by stock weights $w_{P,i}$ this becomes

$$\mathrm{RCI} = \sum_{i=1}^{N} w_{P,i} * \left(\frac{w_{P,i}}{w_{\mathrm{MW},i}}\right) \quad \text{(A2)}$$

This expression is also suggested by Vangelisti (2006) who calls it *portfolio concentration*. We prefer to call it the *Relative Concentration Index* (*RCI*) of a portfolio to distinguish this type of concentration from absolute concentration which is typically a function of the sum of the squares of the portfolio weights or, equivalently, the Herfindahl Index which is a diversification proxy rather than a trading cost proxy.

In matrix format, *RCI* in Equation (A2) can be expressed as $w'M^{-1}w$. Here, M^{-1} is a diagonal square matrix where each element along the diagonal is one divided by the market cap of stock i and the off-diagonal elements are zero. If we add a term, γ, to capture the investor's aversion to trading costs we get $\gamma w'M^{-1}w$.

Using this term as our proxy for trading costs gives

$$U(w) = w'r - \frac{1}{2}\lambda w'\Sigma w - \frac{1}{2}\gamma w'M^{-1}w. \quad \text{(A3)}$$

B. Equal weighted vs market weighted portfolio returns

The portfolio return r_P can be decomposed as follows:

$$r_P = E(r_i) + N \cdot \mathrm{cov}(w_{P,i}, r_i) \quad \text{(B1)}$$

This relationship can be derived as follows:

$$\begin{aligned} r_P &= \sum_{i=1}^{N} w_{P,i} r_i \\ &= N \cdot E(w_{P,i} r_i) \\ &= N \cdot E(w_{P,i}) \cdot E(r_i) + N \cdot \operatorname{cov}(w_{P,i}, r_i) \\ &= E(r_i) + N \operatorname{cov}(w_{P,i}, r_i) \end{aligned}$$

which follows from the definition of covariance, namely $\operatorname{cov}(x, y) = E(xy) - E(x)E(y)$ for variables x and y.

Now, let r_{MW} and r_{EW} be the MW portfolio return and the EW portfolio return of the same assets.

From Equation (B1) and the fact that for an EW portfolio $\operatorname{cov}(w_{P,i}, r_i) = 0$ by construction (hence $r_{\mathrm{EW}} = E(r_i)$), we can derive

$$r_{\mathrm{EW}} - r_{\mathrm{MW}} = -N \operatorname{cov}(w_{\mathrm{MW},i} r_i) \tag{B2}$$

Therefore the difference in return between EW and MW of the same assets is determined by the degree to which asset returns covary with market weights.

Therefore, if returns have a positive (negative) correlation with market weights, MW will have a higher (lower) return than EW.

Evolving weights

We now look at the way that weights evolve in MW. For the sake of simplicity we assume that stocks pay no dividends and we ignore new issuance or companies entering or leaving the market. Therefore in MW, stock weights drift exactly in relation to their relative returns. It is easier to represent this using the natural logarithm (log) of weights and logarithmic returns (upper case W and R). Thus

$$W_{\mathrm{MW},i,t+1} = W_{\mathrm{MW},i,t} + R_{i,t} - R_{\mathrm{MW},t}$$

So the log weight of an asset in MW at $t + 1$ is exactly equal to the previous log weight of the asset plus the difference in the log returns of the asset and the MW portfolio. If we then calculate the cross-sectional variance (denoted by D^2 (…) for the square of the cross-sectional dispersion D (…)) of the log weights at times t and $t + 1$ we have

$$D^2(W_{\mathrm{MW},i,t+1}) = D^2(W_{\mathrm{MW},i,t} + R_{i,t} - R_{\mathrm{MW},t})$$

Since $R_{\mathrm{MW},t}$ is a constant, this is just

$$D^2(W_{\mathrm{MW},i,t+1}) = D^2(W_{\mathrm{MW},i,t} + R_{i,t})$$

which expands to

$$\begin{aligned} D^2(W_{\mathrm{MW},i,t+1}) = {} & D^2(W_{\mathrm{MW},i,t}) + D^2(R_{i,t}) \\ & + 2\operatorname{cov}(W_{\mathrm{MW},i,t}, R_{i,t}) \end{aligned} \tag{B3}$$

where $D^2(R_{i,t})$ is the cross-sectional variance of log returns and $\operatorname{cov}(W_{\mathrm{MW},i,t}, R_{i,t})$ is the cross-sectional covariance of log market weights and log returns.

Rearranging we have

$$\begin{aligned} \operatorname{cov}(W_{\mathrm{MW},i,t}, R_{i,t}) = {} & \frac{1}{2}(D^2(W_{\mathrm{MW},i,t+1}) \\ & - D^2(W_{\mathrm{MW},i,t}) \\ & - D^2(R_{i,t})) \end{aligned}$$

Using the above and with some further algebra, we can approximate the covariance term in Equation (B2) as

$$\begin{aligned} -N \operatorname{cov}(w_{\mathrm{MW},i,t}, R_{i,t}) \approx {} & -\frac{1}{2}(D^2(W_{\mathrm{MW},i,t+1}) \\ & - D^2(W_{\mathrm{MW},i,t}) \\ & - D^2(R_{i,t})) \end{aligned} \tag{B4}$$

We can break down the right-hand side of Equation (B4) into two components.

The first two terms in the brackets on the right-hand side relate to the change in the cross-sectional variance of the log weights. A useful way to represent this is to change it so that it uses a simple measure of diversification. The Herfindahl Index of a portfolio is the sum of the square of the (arithmetic) weights of the assets in the portfolio.

$$H_p = \sum w_{P,i}^2$$

We can show that

$$D^2(W_{i,t+1}) - D^2(W_{i,t}) = \ln(H_{p,t+1}) - \ln(H_{p,t})$$

So the change in the cross-sectional variance in the log weights of a portfolio is the same as the change in the log of the Herfindahl Index of the portfolio which we could represent as simply $\mathrm{d}H_p$.

The second part of the right-hand side of Equation (B4) is $\frac{1}{2}(D^2(R_{i,t}))$.

This is the diversification return of EW as described in Equation (3). We can now modify Equation (B3) to be

$$-N \operatorname{cov}(w_{\mathrm{MW},i,t}, R_{i,t}) \approx -\frac{1}{2}\mathrm{d}H_{\mathrm{MW}} + \mathrm{DR}_{\mathrm{EW}} \tag{B5}$$

In other words, Equation (B5) means that the return difference between EW and MW, which is $-N \operatorname{cov}(w_{\mathrm{MW},i,t}, r_{i,t})$, is related to the negative of the change in the log of the Herfindahl Index of MW plus the diversification return of EW. In general, H_{MW} is observed to be a mean reverting statistic and therefore the long-run expected value of the change in H, i.e. $E(\mathrm{d}H_{\mathrm{MW}})$, is zero. If this is the case, then we can infer that the expected excess return of EW over MW is the expected diversification return of EW, i.e. $\mathrm{DR}_{\mathrm{EW}}$. If MW concentrates and H_{MW} increases, then if $\mathrm{d}H$ is greater than $\mathrm{DR}_{\mathrm{EW}}$, this will lead to an underperformance of EW vs MW. However, $\mathrm{DR}_{\mathrm{EW}}$ is always positive and so eventually EW will outperform MW under any plausible scenario that does not involve the market concentrating excessively in a single stock or sector.

C. Shrinkage

We choose the parameters a and λ from Equation (2) in a way to achieve certain levels of total portfolio shrinkage (TSP) and certain percentage contributions to the total shrinkage from stock and sector shrinkage ($\mathrm{PCS}_{\mathrm{Stock}}$ and $\mathrm{PCS}_{\mathrm{Sec}}$, respectively).[20] From Equation (2) we have

$$\Sigma = \mathrm{aI} + \mathrm{bSS}'$$

This term splits total stock return variability into a stock and a sector component. Moving to a portfolio level we can rewrite this as

$$\begin{aligned}\mathrm{TSS}_P &= w'_P \Sigma w_P \\ &= a w_P' I w_P + b w_P' S S' w_P \\ &= a H_P + b H_{P,\mathrm{Sec}}\end{aligned}$$

where TSS_P is the total sum of squares, H_P and $H_{P,\mathrm{Sec}}$ are the stock and sector level Herfindahl Indices respectively of portfolio P. As a result, the total portfolio shrinkage away from market weights towards equal stock and sector weights can be expressed as

$$\begin{aligned}\mathrm{TSS}_{\mathrm{MW}} - \mathrm{TSS}_P &= a(H_{\mathrm{MW}} - H_P) \\ &\quad + b(H_{\mathrm{MW,Sec}} - H_{P,\mathrm{Sec}}) \\ &= a\Delta H_P + b\Delta H_{P,\mathrm{Sec}}\end{aligned}$$

This term can be broken down into the stock level and sector level percentage contributions ($\mathrm{PCS}_{\mathrm{Stock}}$ and $\mathrm{PCS}_{\mathrm{Sec}}$) to the total portfolio shrinkage ($\mathrm{PCS}_{\mathrm{Stock}}$ and $\mathrm{PCS}_{\mathrm{Sec}}$, respectively) where

$$\mathrm{PCS}_{\mathrm{Stock}} = \frac{a\Delta H_P}{a\Delta H_P + b\Delta H_{P,\mathrm{Sec}}} \quad \text{and}$$

$$\mathrm{PCS}_{\mathrm{Sec}} = \frac{b\Delta H_{P,\mathrm{Sec}}}{a\Delta H_P + b\Delta H_{P,\mathrm{Sec}}}$$

Hence, $\mathrm{PCS}_{\mathrm{Stock}} + \mathrm{PCS}_{\mathrm{Sec}} = 1$. Expressing the total portfolio shrinkage in percentage terms and denoting it by TSP, we get

$$\mathrm{TSP} = \frac{\mathrm{TSS}_{\mathrm{MW}} - \mathrm{TSS}_P}{\mathrm{TSS}_{\mathrm{MW}} - \mathrm{TSS}_{\mathrm{EW}}} = \frac{a\Delta H_P + b\Delta H_{P,\mathrm{Sec}}}{a\Delta H_{\mathrm{EW}} + b\Delta H_{\mathrm{EW,Sec}}}$$

Hence TSP is the total percentage amount of shrinkage in a portfolio away from market weights and towards equal stock and sector weights expressed as a proportion of the total possible shrinkage. $\mathrm{PCS}_{\mathrm{Stock}}$ and $\mathrm{PCS}_{\mathrm{Sec}}$ break up this total shrinkage amount into percentage contributions from the stock and sector components.

Notes

1 See, for example, Black *et al.* (1973), Fama and French (1992), Baker *et al.* (2011), Clare *et al.* (2013) or Arnott *et al.* (2013).

2 A recent article in the Economist illustrates this issue (http://www.economist.com/news/briefing/21586558-american-private-enterprise-dominates-corporate-premier-league-again-thanks-waning).

3 See Appendix A for a derivation of this expression.

4 In order to scale M^{-1} appropriately, we can divide all elements in M^{-1} (where the elements along the diagonal are the reciprocals of stocks' market weights) by their cross-sectional average.

5 For an overview of different alternative indexing strategies see, for example, Chow *et al.* (2011) or Lee (2011).

6 Asness (2006) demonstrates that Fundamental Indexing™ simply overweights stocks that have a higher valuation ratio relative to the valuation ratio of the aggregate market and underweights them otherwise. Market weights remain the "neutral" reference portfolio. As a consequence, Fundamental Indexing™ would be located on the right-hand side of the above triangle. To see this, let F_i be some fundamental such as a stock's total dividend and M_i be its market cap. We can weight a stock relative to its market weight based on its relative dividend yield y_i/y_{MW}, i.e. the yield of the stock divided by the yield of the market. Thus we get $w_{P,i} = w_{\mathrm{MW},i} \times \frac{y_i}{y_{\mathrm{MW}}} = \left(\frac{M_i}{\sum M_i}\right) \times \left(\frac{F_i/M_i}{\sum F_i/\sum M_i}\right)$, where $w_{P,i}$ is the portfolio weight and $w_{\mathrm{MW},i}$ is the market weight. This simply solves to $F_i/\sum F_i$ which is the fundamentally weighted position.

7 We will explore the combination of diversification with return expectations in future research.

8 DeMiguel *et al.* use EW as a *naïve* benchmark for their mean–variance strategies. In fact, as our subsequent discussion on the diversification return demonstrates, EW may not be so naïve after all—a finding which is likely to have implications for their results.

9 Our approach can easily be extended to countries. Both sector and country effects have been shown to be major determinants of individual stock and portfolio risk and are included in commercial risk models.

10 As Europe consists of multiple countries, this gives rise to country effects in addition to sector effects. To keep our analysis simple we ignore country effects. Country effects could, however, be incorporated into our portfolio construction framework in exactly the same way as sector effects.

11 The empirical results are not sensitive to the calendar month chosen for rebalancing. Annual as opposed to more frequent rebalancing lowers portfolio turnover and hence trading costs. It also allows us to avoid inducing a negative exposure to the momentum effect (see Jegadeesh and Titman, 1993) since, for example, an EW portfolio constantly reduces positions in stocks that have drifted up vs the market and vice versa.

12 This can be achieved using a simple numerical optimization routine. The computational details for TSP as well as $\mathrm{PCS}_{\mathrm{Stock}}$ and $\mathrm{PCS}_{\mathrm{Sec}}$ are shown in Appendix C.

13 For the 50/50 blend TSP = 1.0 is infeasible. However, we shrink the 50/50 blend towards equal stock and sector weights as much as possible. We are able to achieve the following average total shrinkage (TSP) levels over time for our 50/50 blends: United States = 0.93, Japan = 0.86, Europe = 0.91.

14 Returns shown are computed as the sum of the regression intercept and the residual return series and then cumulated over time.

15 We multiply the Herfindahl Index by the number of stocks (sectors) in the portfolio as the magnitude of the Herfindahl Index would otherwise not be comparable over time whenever the number of stocks (sectors) in the investment universe changes.

16 Note that although Equation (4) refers to fully equal-weighted portfolios, we expect our diversified portfolios which are blends between market weights and equal stock and sector weights to be strongly exposed to the relative return components shown in Equation (4).

17 The factor returns for SMB, HML and MOM have been obtained from Ken French's Web site: (http://mba.tuck.

dartmouth.edu/pages/faculty/ken.french/data_library.html).

[18] Our diversified portfolios compare similarly to MW indices for earlier snapshots which are not shown here to save space.

[19] Note that we focus on market impact costs rather than bid–ask spreads as the former tend to dominate trading costs for larger trades.

[20] While Keim and Madhavan focus on trades (changes in w) we focus on the level of w as it represents the summation of changes over time.

[21] This can be achieved using a simple numerical optimization routine.

References

Arnott, R., Hsu, J., Kalesnik, J., and Tindall, P. (2013). "The Surprising Alpha from Malkiel's Monkey and Upside-Down Strategies," *Journal of Portfolio Management* **39**(4), 91–105.

Asness, C. (2006). "The Value of Fundamental Indexing," *Institutional Investor* 95–99.

Baker, M., Bradley, B., and Wurgler, J. (2011). "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly," *Financial Analysts Journal* **67**(1), 40–54.

Black, F., Jensen, M., and Scholes, M. (1972). "The Capital Asset Pricing Model: Some Empirical Tests," In: Michael C. Jensen (ed.), *Studies in the Theory of Capital Markets*. Praeger, New York, pp. 79–121.

Booth, D. and Fama, E. (1992). "Diversification Returns and Asset Contributions," *Financial Analysts Journal* **48**(3), 26–32.

Bouchey, P., Nemtchinov, V., Paulsen, A., and Stein, D. (2012). "Volatility Harvesting: Why Does Diversifying and Rebalancing Create Portfolio Growth?" *Journal of Wealth Management* **15**(2).

Carhart, M. (1997). "On Persistence in Mutual Fund Performance," *Journal of Finance* **52**(1), 57–82.

Chouiefaty, Y. and Coignard, Y. (2008). "Towards Maximum Diversification," *Journal of Portfolio Management* **35**(1), 40–51.

Chow, T., Hsu, J., Kalesnik, V., and Little, B. (2011). "A Survey of Alternative Equity Index Strategies," *Financial Analysts Journal* **67**(5), 37–57.

Clare, A., Motson, N., and Thomas, S. (2013). "An Evaluation of Alternative Equity Indices," *Working Paper*, Cass Business School.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). "Optimal vs Naive Diversification: How Inefficient Is the $1/N$ Portfolio Strategy?" *Review of Financial Studies* **22**(5), 1915–1953.

Fama, E. and French, K. (1992). "The Cross-Section of Expected Returns," *Journal of Finance* **47**(2), 427–465.

Fama, E. and French, K. (1993). "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics* **33**, 3–56.

Fernholz, R., Garvy, R., and Hannon, J. (1998). "Diversity-Weighted Indexing," *Journal of Portfolio Management* **24**, 74–82.

Fernholz, R. (2002). *Stochastic Portfolio Theory*, Springer-Verlag, New York.

Jegadeesh, N. and Titman, S. (1993). "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance* **48**(1), 65–91.

Keim, D., and Madhavan, A. (1997). "Transaction Costs and Investment Style: An Inter-Exchange Analysis of Institutional Equity Trades," *Journal of Financial Economics* **46**, 265–292.

Lee, W. (2011). "Risk-Based Asset Allocation: A New Answer to An Old Question?" *Journal of Portfolio Management* **37**(4), 11–28.

Leclerc, F., L'Her, J., Mouakhar, T., and Savaria, P. (2013). "Industry-Based Alternative Equity Indices," *Financial Analysts Journal* **69**(2), 42–56.

Michaud, R. (1989). "The Markowitz Optimization Enigma: Is Optimization Optimal?" *Financial Analysts Journal* **45**(1), 31–42.

Petajisto, A. (2013). "Active Share and Mutual Fund Performance," *Financial Analysts Journal* **69**(4), 73–93.

Plyakha, Y., Uppal, R., and Vilkov, G. (2012). "Why Does an Equal-Weighted Portfolio outperform Value- and Price-Weighted Portfolios," *Working Paper*, Edhec Business School.

Qian, E. (2012). "Diversification Return and Leveraged Portfolios," *Journal of Portfolio Management* **38**(4), 14–25.

Vangelisti, M. (2006). "The Capacity of an Equity Strategy," *Journal of Portfolio Management* **32**(2), 44–50.

Velvadapu, P. (2011). "The Evolution of Equal Weighting," *Journal of Indexes* **14**(1), 20–29.

Willenbrock, S. (2011). "Diversification Return, Portfolio Rebalancing, and the Commodity Return Puzzle," *Financial Analysts Journal* **67**(4), 42–49.

Yu, W. and Sharaiha, Y. (2007). "Alpha Budgeting—Cross-Sectional Dispersion Decomposed," *Journal of Asset Management* **8**(1), 58–72.

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